CORPUS CHRISTI COLLEGE SEQUERE DOMINUM	**Corpus Christi College** **Year 12 Mathematics Applications** **Test 5 – Networks and Decision Maths**

Name: SOLUTIONS

Date: 14th Sept 2021
Time: 45 minutes
Weight: 8%
Total marks: 45 marks

Teacher:

Result

45

TOPICS: Networks and Decision Maths

INSTRUCTIONS:

- Answer the questions in the spaces provided
- Show all necessary working out
- Marks may be deducted for careless or untidy work
- 1 A4 Page of notes (both sides) allowed
- Calculators are allowed

Student Reflection

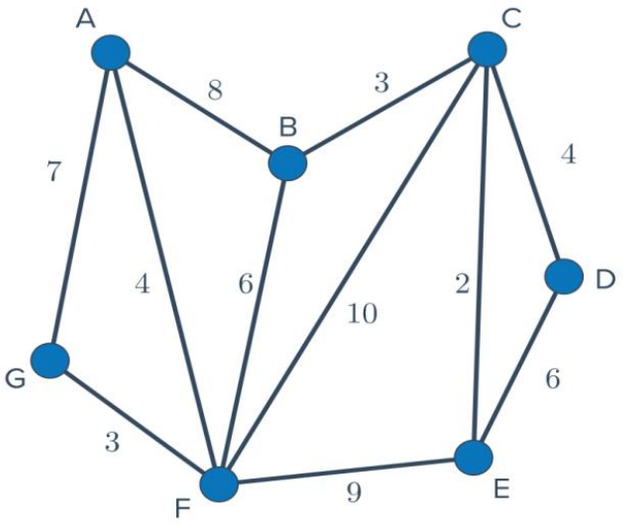
Qu 1	Qu 2, 3, 5	Qu 4	Qu 6	Qu 7	Qu 8	Qu 9	TOTAL
Minimum spanning tree - graph	Maximum Flow	Minimum Spanning Tree - table	Project networks	Constructing a project network	Project networks	Hungarian Algorithm	
4	**7**	**9**	**7**	**7**	**3**	**8**	**45**

What Went Well:
I did well at…

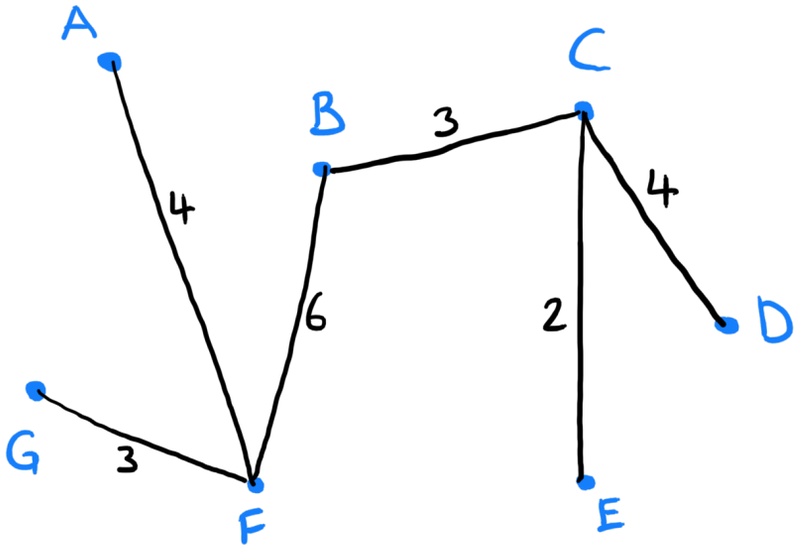
Areas for Development:
I need to improve…

Question 1 **[4 marks]**

a) Re-draw the given diagram as a minimum spanning tree. (3 marks)



Each branch correct ✓✓
(-1 per error)



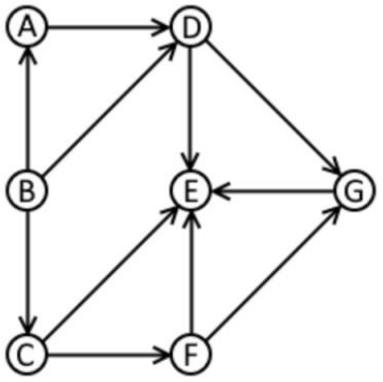
Weights included ✓

b) State the minimum length. (1 mark)

22 units ✓ (allow F/T)

Question 2 **[2 marks]**

For the following flow diagram, identify the vertex that represents the source and the vertex that represents the sink.

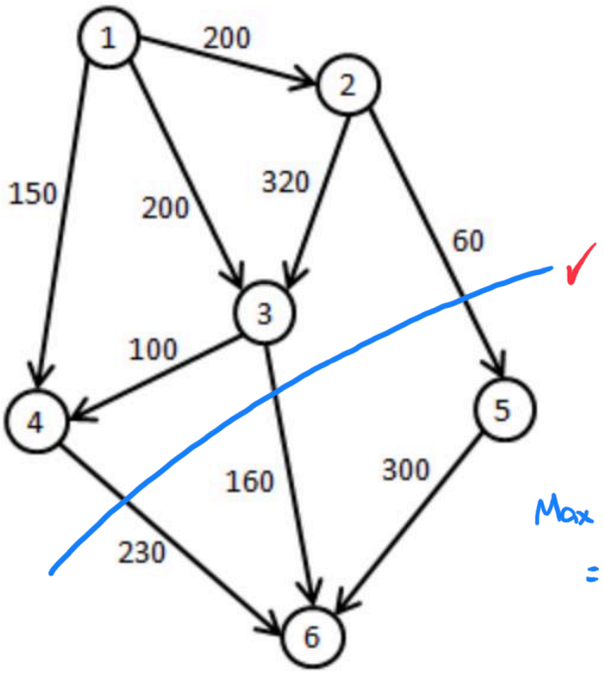


Source : B ✓

Sink : E ✓

Question 3 **[2 marks]**

Using the Minimum Cut Theorem, identify the Maximum Flow of the following network. Clearly display where your minimum cut takes place.



Max Flow / Min Cut = 450 ✓

Question 4 **[9 marks]**

Seven stalls are being set-up for Leavers 2021.The following table shows the lengths in **metres** of the existing paths between each of the stalls A, B, C, D, E, F and G.

	A	*B*	*C*	*D*	*E*	*F*	*G*
A	–	61	64	99	65	79	72
B	61	–	62	79	69	94	53
C	64	62	–	90	91	99	55
D	99	79	90	–	67	78	81
E	65	69	91	67	–	75	80
F	79	94	99	78	75	–	87
G	72	53	55	81	80	87	–

a) Use Prim's algorithm on the table above to calculate the length of the minimum spanning tree for the network of path connecting these stalls. (4 marks)

376 m ✓

✓✓✓ (-1 per error)

(61+53+55+65+67+75)

*The cost of installing a temporary path between stalls is usually **$40/metre.** This year due to lower demand, there is a 15% discount on all temporary paths.*

b) Calculate the minimum cost of installing a set of paths connecting all the stalls (the minimum spanning tree). (3 marks)

$40 × 0·85 = $34 ✓

∴ 34 ✓ × 376 = 12,784

Minimum cost of path $12,784 ✓ (allow F/T)

c) If it is found that the path between stalls E and F can no longer be used due to it being blocked by a rave tent. There still needs to be a path connected to F. How does this change impact the overall length of the pathway? State the total new length of path. (2 marks)

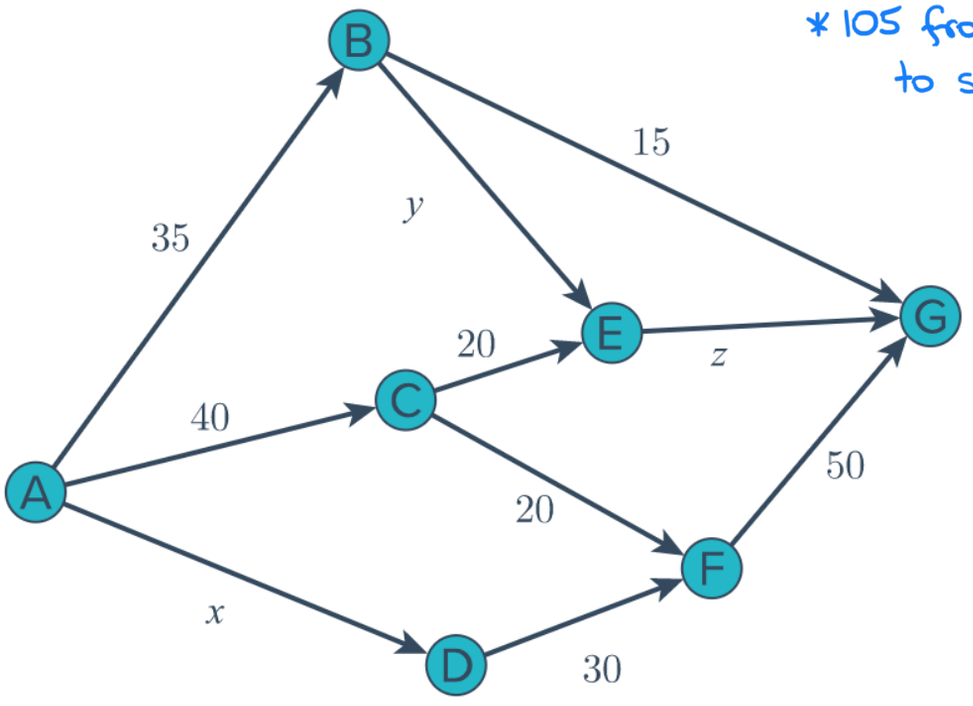
EF = 75m and DF = 78m

∴ 3 metre increase ✓

New total is 379m ✓

Question 5 **[3 marks]**

The following diagram shows the flow of water along each pipe when maximum flow operates between the source at A and the sink at G.
Identify the value of the variables.



* 105 from source to sink.

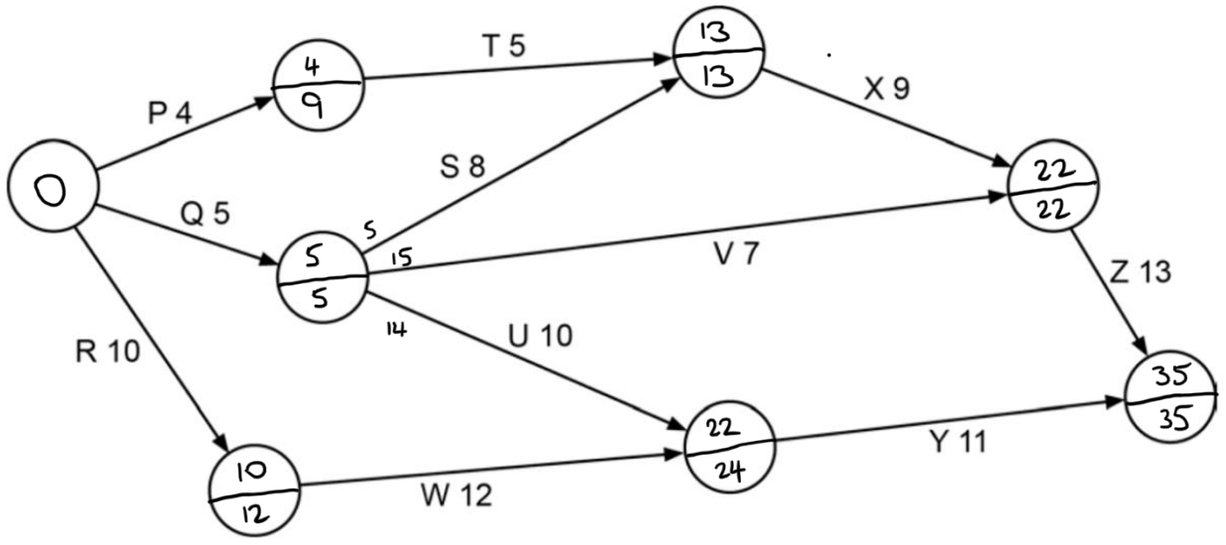
$x = 30$ ✓

$y = 20$ ✓

$z = 40$ ✓

Question 6 **[7 marks]**

A project consists of 11 activities, P to Z. The project network representing the scheduling of these activities is shown below. The times are in days.



a) State the critical path and the minimum completion time for this project (2 marks)

QSXZ ✓

35 days ✓

b) Determine the earliest starting time for activity Y (1 mark)

Day 22 ✓

c) Determine the latest starting time for activity V (1 mark)

Day 15 ✓

d) Determine the float time for activity U (1 mark)

9 days ✓

e) Activity W is delayed by three days. How, if at all, will this affect the critical path and minimum completion time for this project? (2 marks)

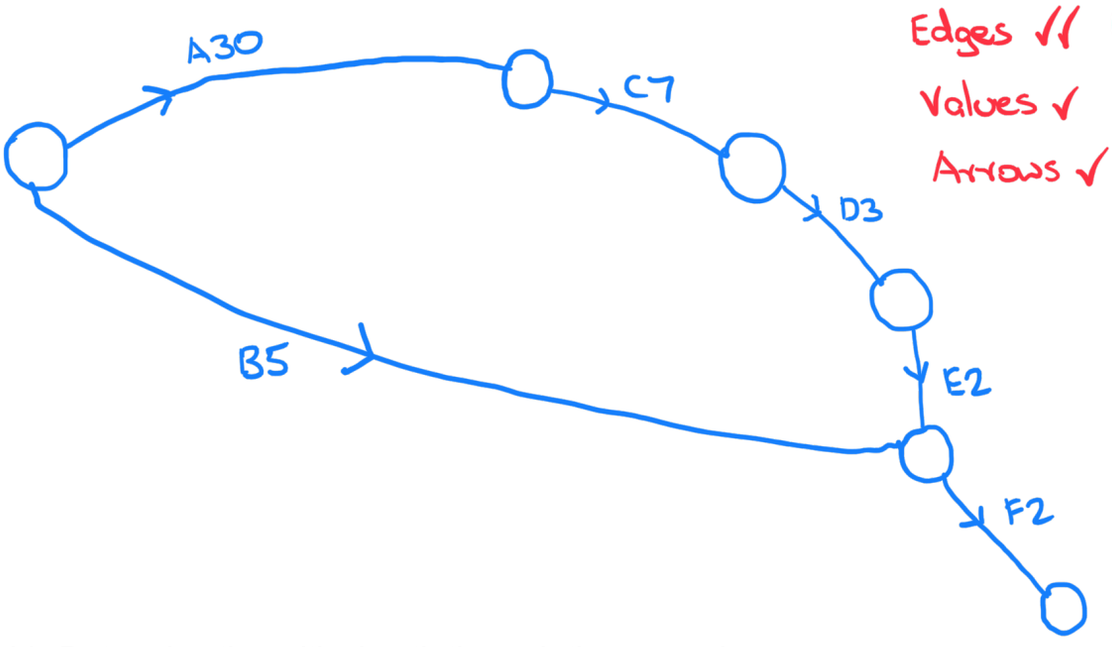
- Critical path changes to RWY ✓
- Minimum time increases by one to 36 ✓

Question 7 **[7 marks]**

To start a fire while camping, the following steps should be taken.

Activity	Description	Dependencies	Duration (mins)
A	Gather logs, twigs, and dried leaves.	-	30
B	Obtain lighter or matches.	-	5
C	Pile up the dried leaves at the bottom.	A	7
D	Cover dried leaves with twigs.	C	3
E	Place two logs at the side of the pile and one log across	D	2
F	Light up the pile from the bottom.	B, E	2

a) Sketch the project network that represents the above activity (4 marks)



Edges ✓✓ (-1 per error)
Values ✓
Arrows ✓

b) Determine the critical path through the network (1 mark)

ACDEF ✓

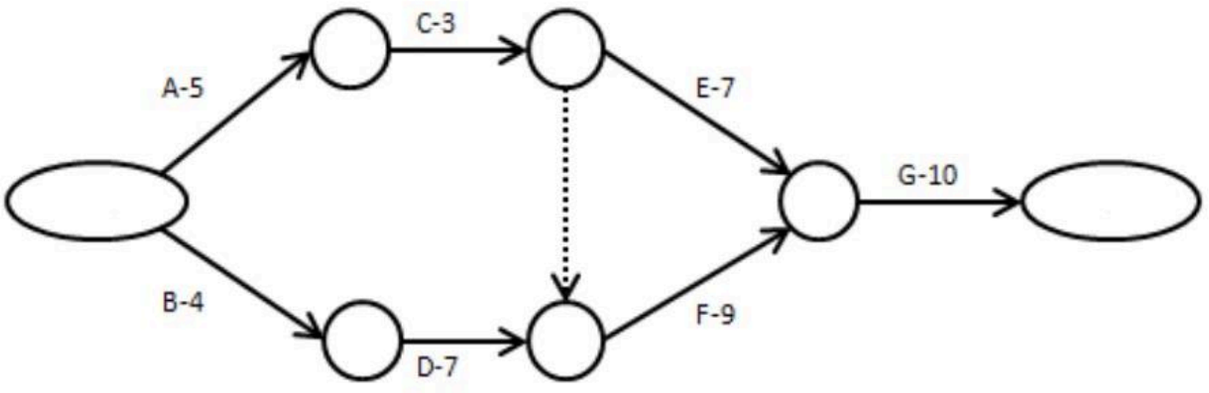
c) If finding a lighter or matches took 15 minutes longer, how would that affect the time taken to start the fire? Justify your decision. (2 marks)

• No change in time, fire is lit on time. ✓

Any valid reason:
- Not on critical path
- Plenty of slack time ✓

Question 8 **[3 marks]**

A project network is given below in minutes.



a) Determine the minimum completion time of this project. (1 mark)

30 minutes ✓

b) List the immediate predecessor/s of activity F. Briefly justify your choice. (2 marks)

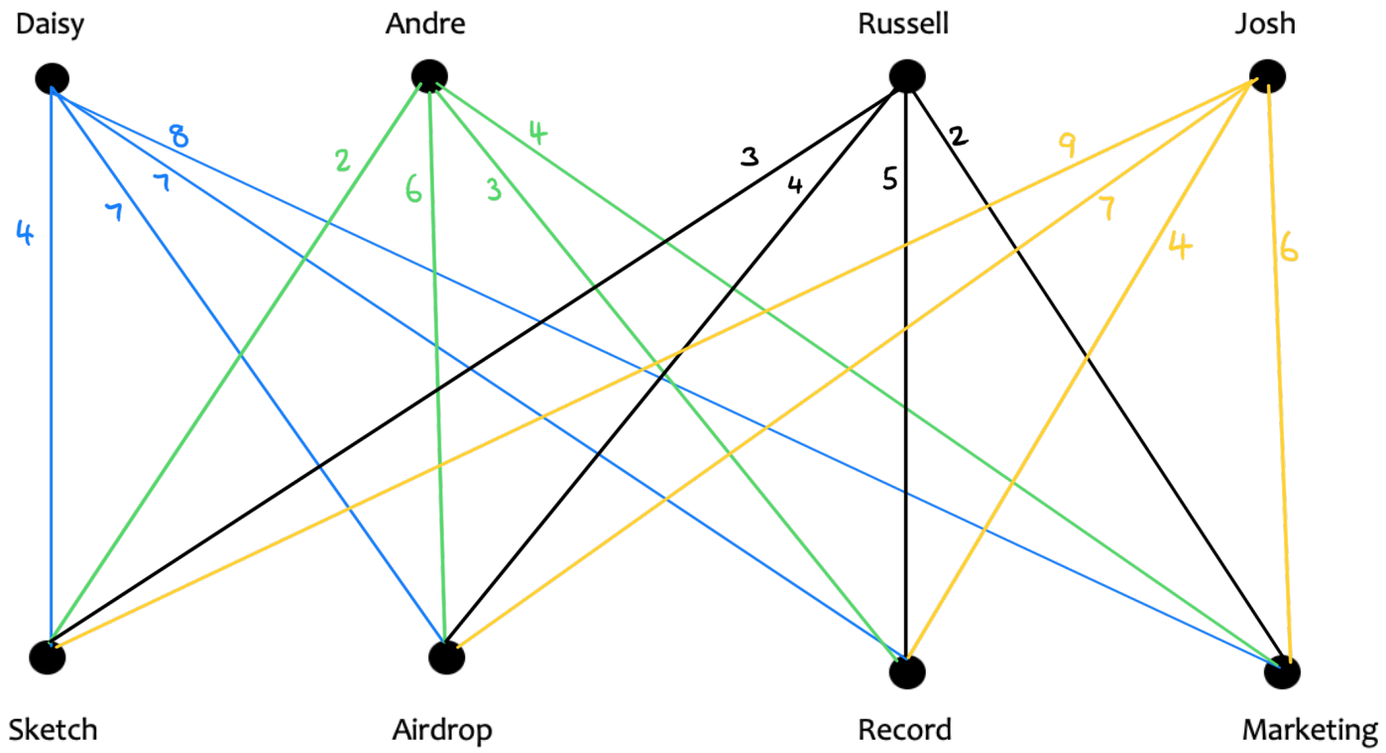
- C and D ✓
- Indicates use of dummy path ✓

Question 9 **[8 marks]**

Leah has four workers in her business, Daisy, Andre, Russell and Josh and four jobs to complete. The time, in hours, that each worker can complete a particular job is given in the below table.

	Sketch	Airdrop	Record	Marketing
Daisy	4	7	7	8
Andre	2	6	3	4
Russell	3	4	5	2
Josh	9	7	4	6

a) Represent this information as a weighted Bipartite Graph (2 marks)



Complete weighted Bipartite Graph ✓✓

(-1 per error or omission)

b) Using the Hungarian Algorithm, determine which job Leah should assign to each of her workers so that total time is minimised. Show clear workings. (6 marks)

$$\begin{bmatrix} 4 & 7 & 7 & 8 \\ 2 & 6 & 3 & 4 \\ 3 & 4 & 5 & 2 \\ 9 & 7 & 4 & 6 \end{bmatrix} \begin{matrix} -4 \\ -2 \\ -2 \\ -4 \end{matrix}$$

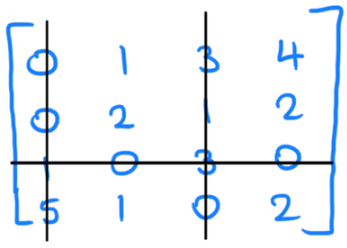
✓ Subtracts lowest values from rows

$$\begin{bmatrix} 0 & 3 & 3 & 4 \\ 0 & 4 & 1 & 2 \\ 1 & 2 & 3 & 0 \\ 5 & 3 & 0 & 2 \end{bmatrix}$$

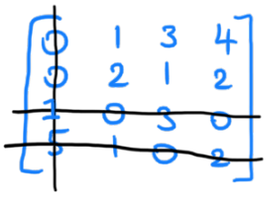
-0 -2 -0 -0

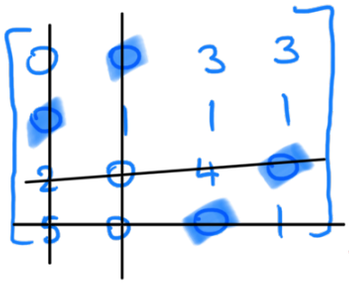
✓ Subtracts lowest values from columns

Alternative



✓ Identifies Solution doesn't exist as
3 Lines ≠ 4 Matrix Order





✓ -1 from numbers with a Line
+1 to numbers at Intersection

✓ Identifies Solution is possible as
4 Lines = 4 Matrix Order

Andre ; sketch

Daisy ; Airdrop

Russell ; Market

Josh ; Record

✓ Correctly assigns jobs

End of Test ☺